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INFERENCE ON THE COMMON COEFFICIENT OF VARIATION WHEN POPULATIONS ARE LOGNORMAL: A SIMULATION-BASED APPROACH

C. K. NG

Department of Management Sciences
City University of Hong Kong
Hong Kong
P. R. China
e-mail: ms8sf344@cityu.edu.hk

Abstract

The coefficient of variation is often used as a measure of precision in medical and biological sciences. When it is known a priori that several independent normal populations have equal coefficient of variations, procedures for constructing confidence intervals for the common coefficient of variation based on the concept of generalized variables had been discussed by other researchers. This paper makes use of the same concept applied to lognormal populations.

1. The Lognormal Distribution

A random variable is said to have the lognormal distribution if the logarithm of that variable has a normal distribution. A lognormal distribution results if the variable is the product of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the sum of a large number of independent, identically distributed variables. A lognormal random variable X with parameters μ_L, σ_L has the probability density function

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$$f(x;\,\mu_L,\,\sigma_L) = \frac{1}{x\sigma_L\sqrt{2\pi}}\exp\left[-\left(\frac{\ln(x)-\mu_L}{\sigma_L\sqrt{2}}\right)^2\right] \quad (x > 0,\ \sigma_L > 0).$$

The coefficient of variation (CV) of X equals $\sqrt{e^{\sigma_L^2}-1}$, which depends on σ_L only. Albeit less well known than the normal distribution the lognormal, nevertheless, appears everywhere in nature. Examples of quantities which have approximate lognormal distributions include the size of silver particles in a photographic emulsion, the survival time of bacteria in disinfectants, the weight and blood pressure of humans, and the number of words written in sentences by G. B. Shaw. In reliability analysis, the lognormal distribution is often used to model times to repair a maintainable system.

The lognormal distribution also appears very frequently in clinical sciences. Altman & Martin [2] noted that relative risks and odds ratios are often analyzed on a logarithmic scale primarily because the transformations offer distributions, which are closer to normality. Thie et al. [12] stated in their meta-analysis of oncology investigations that numerous biological markers follow non-normal distributions and subsequently require a logarithmic transformation before further analysis. The distribution of the incubation periods for infectious and neoplastic diseases originating from point-source exposures and for genetic diseases follow a lognormal distribution, the so called Sartwell's model. Uptake of radioactivity of a population of cells labelled with ^{210}Po was also found to be well described by a lognormal distribution, see Neti & Howell [9].

2. The Coefficient of Variation

The coefficient of variation (CV), which is the ratio of the standard deviation to the mean, is a dimensionless measure of dispersion found to be very useful in many situations. In chemical experiments, the CV is

often used as a yardstick of precision of measurements; two measurement methods may be compared on the basis of their respective CVs. In finance, the CV can be used as a measure of relative risks (Miller & Karson [8]) and a test of the equality of the CVs for two stocks can help determine if the two stocks possess the same risk or not. Hamer et al. [7] used the CV to assess homogeneity of bone test samples produced from a particular method to help assess the effect of external treatments, such as irradiation, on the properties of bones. Ahn [1] used the CV in the analysis of fault trees. The CV has also been employed by Gong & Li [6] in assessing the strength of ceramics. Sometimes, it might be easier to work with the reciprocal of the CV, denoted ICV. The ICV has special applications in parametric inference problems for some important lifetime distributions. Several methods for constructing the confidence interval of the lognormal mean are studied in Zhou & Gao [17] and Olsson [10]. However, apart from Pang et al. [11], who make use of MCMC simulation for the construction of confidence intervals for the lognormal mean, relatively little has been done in this area for the lognormal CV. Tian [13] is another one who uses simulation technique to develop inference procedures for the common CV based on independent normal samples. This paper follow her approach with independent normal samples replaced by lognormal samples.

3. The Generalized Variable Approach

3.1. Generalized *p*-value, generalized pivot and generalized confidence interval

The generalized *p*-value was introduced by Tsui & Weerahandi [14] and the generalized confidence interval by Weerahandi [15]. When traditional interval estimation and hypothesis testing procedures do not exist, these concepts provide confidence intervals and *p*-values for hypothesis testing. To fix ideas, suppose that $\underset{\sim}{X} = (X_1, X_2, \ldots, X_n)$ is a random sample from a distribution which depends on a vector of

parameters $\underset{\sim}{\theta} = (\psi, \underset{\sim}{\nu})$, where ψ is the parameter of interest and $\underset{\sim}{\nu}$ is a vector of 'nuisance' parameters. Suppose a confidence interval for ψ is desired. Weerahandi [15] defines a generalized pivot $R(\underset{\sim}{X}, \underset{\sim}{x}, \psi, \underset{\sim}{\nu})$ for this problem, where $\underset{\sim}{x}$ is an observed value of $\underset{\sim}{X}$, as a random variable having the following two properties:

(1) $R(\underset{\sim}{X}, \underset{\sim}{x}, \psi, \underset{\sim}{\nu})$ has a distribution free of the vector of nuisance parameters $\underset{\sim}{\nu}$,

(2) The observed value of $R(\underset{\sim}{X}, \underset{\sim}{x}, \psi, \underset{\sim}{\nu})$ is ψ.

Suppose $R(\alpha)$ denotes the 100α-th percentile of the distribution of R. Then the $100(1-\alpha)\%$ two-sided generalized confidence interval for ψ is given by $(R(\alpha/2), R(1-\alpha/2))$. To give a specific example, let us consider a random sample of size n from a normal population with mean μ and variance σ^2. Let $\bar{x}$ be the observed value of $\bar{X}$ and s^2 be the observed value of S^2. It can easily be seen that the generalized pivot for the ICV $\theta := \mu/\sigma$, given in Weerahandi [16], is

$$R_\theta := \frac{\bar{x}}{s}\frac{S}{\sigma} - \frac{\bar{X}-\mu}{\sigma} = \frac{\bar{x}}{s}\sqrt{\frac{U}{n-1}} - \frac{Z}{\sqrt{n}},$$

where $Z = \dfrac{\sqrt{n}(\bar{X}-\mu)}{\sigma} \sim N(0,1)$ and $U = \dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$.

Obviously, the observed value of R_θ is θ and its distribution does not depend on nuisance parameters. Therefore, R_θ is a generalized pivot for θ. The corresponding generalized pivot for the CV, ψ, is $R_\psi := 1/R_\theta$. In accord with the previous notation, $\underset{\sim}{\nu} = (\mu, \sigma)$.

Next consider the testing problem $H_0 : \psi = \psi_0$ vs $H_1 : \psi < \psi_0$. A generalized test variable $T(\underset{\sim}{X}, \underset{\sim}{x}, \psi, \underset{\sim}{\nu})$ is a random variable that satisfies the following three conditions (Tsui & Weerahandi [14]):

(1) For fixed $\underset{\sim}{x}$, the distribution of $T(\underset{\sim}{X}, \underset{\sim}{x}, \psi, \underset{\sim}{\nu})$ is free of the vector of nuisance parameters $\underset{\sim}{\nu}$.

(2) The value of $T(\underset{\sim}{X}, \underset{\sim}{x}, \psi, \underset{\sim}{\nu})$ at $\underset{\sim}{X} = \underset{\sim}{x}$ is free of $\underset{\sim}{\nu}$.

(3) For fixed $\underset{\sim}{x}$ and $\underset{\sim}{\nu}$ and for all t, $\Pr[T(\underset{\sim}{X}, \underset{\sim}{x}, \psi, \underset{\sim}{\nu}) \geq t]$ is either an increasing function or a decreasing function of ψ.

A generalized critical region is defined to be

$$\{\underset{\sim}{X} : T(\underset{\sim}{X}, \underset{\sim}{x}, \psi, \underset{\sim}{\nu}) \geq \text{or} \leq T(\underset{\sim}{x}, \underset{\sim}{x}, \psi, \underset{\sim}{\nu})\},$$

depending on whether $T(\underset{\sim}{X}, \underset{\sim}{x}, \psi, \underset{\sim}{\nu})$ is stochastic increasing or decreasing in ψ. The generalized p-value is then $\Pr[T(\underset{\sim}{X}, \underset{\sim}{x}, \psi, \underset{\sim}{\nu}) \geq \text{or} \leq T(\underset{\sim}{x}, \underset{\sim}{x}, \psi, \underset{\sim}{\nu}) \mid \psi_0]$. If we want to test $H_0 : \psi = \psi_0$ against $H_1 : \psi < \psi_0$, the generalized test variable would be $T_\psi = R_\psi - \psi$, which obviously satisfies all three conditions. In the following, we will develop the generalized pivot and generalized test variable for the common CV from several independent populations.

3.2. The proposed approach

For $i = 1, 2, \ldots, k$, let ψ be the common population CV, $\bar{x}_i$ the observed value of $\overline{X}_i$ and s_i^2 the observed value of S_i^2 both computed from a sample of size n_i. The generalized pivot for $\theta := 1/\psi$ is

$$R_\theta^i := \frac{\bar{x}_i}{s_i}\frac{S_i}{\sigma_i} - \frac{\overline{X}_i - \mu_i}{\sigma_i} = \frac{\bar{x}_i}{s_i}\sqrt{\frac{U_i}{n_i - 1}} - \frac{Z_i}{\sqrt{n_i}}. \tag{1}$$

The generalized pivot for ψ based on the i-th sample would then be $R_\psi^i = 1/R_0^i$. If populations are lognormal we have, asymptotically,

$$Z_i = \frac{\sqrt{n_i}(\overline{X}_i - \mu_i)}{\sigma_i} \sim N(0, 1),$$

and

$$U_i = \frac{(n_i - 1)S_i^2}{\sigma_i^2} \sim N(n_i - 1, 2(n_i - 1)).$$

If populations are normal, Feltz & Miller [4] has been proved that, asymptotically,

$$\text{var}\left(\frac{S_i}{\overline{X}_i}\right) = (n_i - 1)^{-1}\psi^2\left(\psi^2 + 0.5\right),$$

which is the formula used by Tian [13]. A similar asymptotic formula which works for normal and non-normal populations alike was later given by Curto & Pinto [3]:

$$\text{var}\left(\frac{S_i}{\overline{X}_i}\right) = n_i^{-1}\psi^2\left(\psi^2 + 0.5\right). \tag{2}$$

A reasonable generalized pivot R_ψ for ψ would then be a weighted average of the individual R_ψ^i's, i.e.,

$$R_\psi = \frac{\sum_{i=1}^k w_i R_\psi^i}{\sum_{i=1}^k w_i},$$

where w_i is taken to be the reciprocal of the estimated $\text{var}\left(\frac{S_i}{\overline{X}_i}\right)$ which, using (2), since our populations are now lognormal, gives

$$R_\psi = \frac{\sum_{i=1}^{k} n_i R_\psi^i}{\sum_{i=1}^{k} n_i w_i}. \tag{3}$$

It is easy to see that the value of R_ψ at the observed value $(\bar{x}_i, s_i)$ is ψ for $i = 1, 2, \ldots, k$. Furthermore, the distribution of R_ψ is independent of nuisance parameters. Therefore, R_ψ is a generalized pivot for constructing confidence intervals of ψ and its quantiles can be used to compute confidence limits. For example, the two-sided $100(1 - \alpha)\%$ confidence interval is given by

$$\big(R_\psi(\alpha / 2),\ R_\psi(1 - \alpha / 2)\big), \tag{4}$$

and the upper one-sided $100(1 - \alpha)\%$ confidence interval is given by

$$\big(-\infty,\ R_\psi(1 - \alpha)\big). \tag{5}$$

These confidence limits depend on the sampling distribution of $\big(\bar{X}_i, S_i\big)$ which, in turn, depends on the parameters (μ_i, σ_i). It is therefore not possible to evaluate the performance of these confidence intervals by analytic methods and simulation has to be used.

The generalized p-value for testing $H_0 : \psi \geq \psi_0$ vs $H_1 : \psi < \psi_0$ is given by

$$P\big(T_\psi \leq 0 \,|\, \psi = \psi_0\big) = P\big(R_\psi \leq \psi_0\big).$$

Similarly, the generalized p-value for testing $H_0 : \psi \leq \psi_0$ vs $H_1 : \psi > \psi_0$ is given by

$$P\big(T_\psi \geq 0 \,|\, \psi = \psi_0\big) = P\big(R_\psi \geq \psi_0\big).$$

Finally, for testing $H_0 : \psi = \psi_0$ vs $H_1 : \psi \neq \psi_0$, it is $2 \min\big[P\big(R_\psi \leq \psi_0\big),\ P\big(R_\psi \geq \psi_0\big)\big]$.

3.3. Simulation algorithm

For a given data set $\{x_{ij}\}$ where $i = 1(1)k$ and $j = 1(1)n_i$, the generalized confidence intervals and the generalized p-values can be computed by the following steps:

1. For $i = 1(1)k$, compute $\bar{x}_i$ and s_i^2.

2. Generate $Z_i \sim N(0, 1)$ and $U_i \sim N(n_i - 1, 2(n_i - 1))$.

3. Compute R_ψ^i for $i = 1(1)k$ using (1) and then R_ψ using (3).

4. Repeat steps 2 and 3 m times to obtain an array of R_ψ's.

5. Arrange this array in ascending order to obtain a new array A_ψ.

To construct confidence intervals, we make use of (4) and (5) and estimate $R_\psi(\alpha)$ by the 100α-th smallest entry of A_ψ.

4. Simulation Study

A simulation program written in R is used to evaluate the coverage probabilities of the proposed confidence intervals and type-I error control of the proposed tests. The CV rarely exceeds 0.50 for most medical and biological studies. Following Fung & Tsang [5] and Tian [13], the common lognormal CV is chosen to be 0.05, 0.10, 0.15, 0.20, 0.30, 0.40, and 0.50. Three or five samples with sample sizes 10, 20, 30, 40, and 50 are considered. Since the lognormal CV, which equals $\sqrt{\exp(\sigma_L^2) - 1}$, is independent of μ_L, all populations are given the same value of μ_L to simplify matters. We take the CV to be 0.05, 0.10, 0.15, 0.20, 0.30, 0.40, and 0.50. For each (μ_L, σ_L) combination, 5000 random samples are generated. With each of these 5000 random samples, 2500 R_ψ's are simulated by the algorithm in Subsection 3.3. Generalized confidence intervals are constructed with a 95% confidence level.

To facilitate comparison with Tian's [13] results, Table I presents the coverage probabilities of one-sided and two-sided confidence intervals for selected normal populations. The coverage probabilities of the proposed confidence intervals are reasonably close to the nominal level regardless of the sample sizes, the number of samples and the value of the common CV. These results agree well with those of Tian's.

Table I. Empirical coverage probabilities and average length of approximate 95% two-sided confidence intervals and empirical coverage probabilities of 95% lower one-sided confidence intervals based on 5000 normal samples

Sample size									
	10			20			30		
C. V.	$CV^{\star}$	Length	$CV^{\star\star}$	$CV^{\star}$	Length	$CV^{\star\star}$	$CV^{\star}$	Length	$CV^{\star\star}$
3 samples									
0.05	0.954	0.033	0.983	0.948	0.020	0.970	0.954	0.016	0.969
0.1	0.956	0.067	0.980	0.953	0.041	0.971	0.951	0.032	0.961
0.15	0.959	0.102	0.980	0.953	0.062	0.968	0.950	0.048	0.967
0.2	0.952	0.142	0.981	0.951	0.085	0.973	0.950	0.066	0.966
0.3	0.950	0.237	0.981	0.956	0.136	0.980	0.951	0.105	0.968
0.4	0.960	0.368	0.984	0.954	0.198	0.979	0.947	0.151	0.976
0.5	0.957	0.570	0.990	0.946	0.278	0.978	0.957	0.207	0.978

Sample size						
	40			50		
C. V.	$CV^{\star}$	Length	$CV^{\star\star}$	$CV^{\star}$	Length	$CV^{\star\star}$
3 samples						
0.05	0.946	0.013	0.967	0.945	0.012	0.961
0.1	0.950	0.027	0.969	0.947	0.024	0.959
0.15	0.947	0.041	0.965	0.949	0.036	0.962
0.2	0.948	0.056	0.971	0.947	0.049	0.963
0.3	0.947	0.089	0.960	0.950	0.078	0.965
0.4	0.951	0.127	0.964	0.945	0.111	0.969
0.5	0.952	0.172	0.975	0.948	0.150	0.968

Sample size									
	10			20			30		
C. V. 5 samples	CV⋆	Length	CV⋆⋆	CV⋆	Length	CV⋆⋆	CV⋆	Length	CV⋆⋆
0.05	0.947	0.026	0.987	0.947	0.016	0.977	0.950	0.012	0.972
0.1	0.941	0.052	0.988	0.936	0.032	0.980	0.948	0.025	0.977
0.15	0.940	0.080	0.990	0.951	0.048	0.983	0.949	0.038	0.975
0.2	0.944	0.111	0.989	0.951	0.066	0.983	0.946	0.051	0.977
0.3	0.935	0.187	0.993	0.946	0.106	0.984	0.944	0.081	0.975
0.4	0.945	0.294	0.993	0.945	0.155	0.985	0.944	0.117	0.975
0.5	0.945	0.471	0.995	0.944	0.218	0.986	0.941	0.161	0.986

Sample size						
	40			50		
C. V. 5 samples	CV⋆	Length	CV⋆⋆	CV⋆	Length	CV⋆⋆
0.05	0.951	0.010	0.970	0.950	0.009	0.968
0.1	0.948	0.021	0.969	0.949	0.019	0.967
0.15	0.944	0.032	0.967	0.952	0.028	0.971
0.2	0.948	0.043	0.972	0.950	0.038	0.970
0.3	0.948	0.069	0.976	0.953	0.061	0.976
0.4	0.943	0.098	0.975	0.945	0.086	0.978
0.5	0.948	0.134	0.978	0.950	0.117	0.979

⋆ Two-sided 95% confidence interval.

⋆⋆ One-sided 95% confidence interval.

Table II gives the empirical coverage probabilities for lognormal samples. When the sample size is 10 or 20, the intervals are rather conservative. For larger sample sizes, the coverage probabilites fall close to the nominal 95%. Understandably, this is so, since formula (2) works well under large samples only.

Table II. Empirical coverage probabilities and average length of approximate 95% two-sided confidence intervals and empirical coverage probabilities of 95% lower one-sided confidence intervals based on 5000 lognormal samples

Sample size									
	10			20			30		
C. V. 3 samples	CV*	Length	CV**	CV*	Length	CV**	CV*	Length	CV**
0.05	0.997	0.405	1.000	0.963	0.027	0.993	0.964	0.018	0.986
0.1	0.997	0.534	1.000	0.959	0.055	0.993	0.958	0.037	0.983
0.15	0.998	0.647	1.000	0.964	0.084	0.993	0.960	0.057	0.984
0.2	0.999	0.755	1.000	0.966	0.114	0.993	0.967	0.077	0.984
0.3	0.998	0.987	1.000	0.967	0.182	0.991	0.957	0.123	0.980
0.4	1.000	1.281	1.000	0.973	0.261	0.989	0.954	0.177	0.971
0.5	1.000	1.651	1.000	0.977	0.365	0.981	0.961	0.245	0.961

Sample size						
	40			50		
C. V. 3 samples	CV*	Length	CV**	CV*	Length	CV**
0.05	0.956	0.015	0.980	0.956	0.013	0.976
0.1	0.954	0.032	0.980	0.954	0.026	0.974
0.15	0.956	0.046	0.976	0.959	0.039	0.975
0.2	0.954	0.062	0.974	0.953	0.053	0.971
0.3	0.956	0.099	0.973	0.948	0.085	0.966
0.4	0.954	0.142	0.962	0.948	0.123	0.955
0.5	0.948	0.197	0.956	0.934	0.169	0.941

Sample size									
	10			20			30		
C. V. 5 samples	CV⋆	Length	CV⋆⋆	CV⋆	Length	CV⋆⋆	CV⋆	Length	CV⋆⋆
0.05	1.000	0.781	1.000	0.965	0.022	0.998	0.960	0.015	0.989
0.1	1.000	0.838	1.000	0.966	0.045	0.997	0.961	0.029	0.989
0.15	1.000	0.909	1.000	0.962	0.068	0.997	0.961	0.045	0.989
0.2	1.000	0.985	1.000	0.962	0.093	0.996	0.960	0.061	0.988
0.3	1.000	1.176	1.000	0.973	0.148	0.994	0.961	0.096	0.983
0.4	1.000	1.409	1.000	0.973	0.215	0.991	0.957	0.140	0.978
0.5	1.000	1.716	1.000	0.978	0.302	0.992	0.968	0.194	0.969

Sample size						
	40			50		
C. V. 5 samples	CV⋆	Length	CV⋆⋆	CV⋆	Length	CV⋆⋆
0.05	0.961	0.012	0.985	0.954	0.010	0.982
0.1	0.959	0.023	0.983	0.955	0.020	0.977
0.15	0.961	0.036	0.982	0.956	0.031	0.981
0.2	0.958	0.048	0.983	0.949	0.042	0.974
0.3	0.946	0.077	0.968	0.947	0.066	0.970
0.4	0.945	0.111	0.965	0.942	0.096	0.957
0.5	0.948	0.154	0.953	0.947	0.133	0.947

⋆ Two-sided 95% confidence interval.

⋆⋆ One-sided 95% confidence interval.

Table III presents simulation results on empirical type-I errors for testing $\psi = \psi_0$ against $\psi \neq \psi_0$ (two-sided) and testing $\psi = \psi_0$ against $\psi < \psi_0$ (one-sided). Overall, the type-I errors for these tests are close to the nominal level when the sample size is sufficiently large.

Table III. Empirical estimates of type-I error at 5% significance level (two-tailed test or left tailed test) based on 5000 samples

Sample size						
	10		20		30	
C. V. 3 samples	2-sided	1-sided	2-sided	1-sided	2-sided	1-sided
0.05	0.004	0.038	0.036	0.062	0.037	0.06
0.1	0.003	0.034	0.037	0.062	0.038	0.063
0.15	0.002	0.036	0.035	0.064	0.039	0.064
0.2	0.002	0.027	0.033	0.06	0.038	0.064
0.3	0	0.023	0.029	0.052	0.038	0.059
0.4	0	0.016	0.024	0.043	0.038	0.053
0.5	0	0.006	0.022	0.04	0.041	0.056

Sample size				
	40		50	
C. V. 3 samples	2-sided	1-sided	2-sided	1-sided
0.05	0.037	0.057	0.041	0.06
0.1	0.045	0.068	0.044	0.063
0.15	0.043	0.064	0.04	0.06
0.2	0.045	0.062	0.045	0.063
0.3	0.048	0.066	0.047	0.062
0.4	0.048	0.055	0.052	0.059
0.5	0.06	0.064	0.06	0.063

Sample size						
	10		20		30	
C. V. 5 samples	2-sided	1-sided	2-sided	1-sided	2-sided	1-sided
0.05	0	0.02	0.036	0.07	0.039	0.07
0.1	0	0.014	0.041	0.071	0.036	0.061
0.15	0	0.014	0.038	0.074	0.041	0.069
0.2	0	0.013	0.04	0.065	0.041	0.073
0.3	0	0.009	0.03	0.058	0.034	0.063
0.4	0	0.002	0.027	0.051	0.038	0.058
0.5	0	0	0.019	0.038	0.036	0.053

Sample size				
	40		50	
C. V. 5 samples	2-sided	1-sided	2-sided	1-sided
0.05	0.045	0.07	0.04	0.061
0.1	0.043	0.068	0.044	0.064
0.15	0.042	0.068	0.05	0.074
0.2	0.041	0.064	0.049	0.07
0.3	0.041	0.058	0.048	0.066
0.4	0.043	0.063	0.05	0.06
0.5	0.047	0.057	0.062	0.064

5. An Example

The Hong Kong Medical Technology Association has been conducting the Quality Assurance Programme for medical laboratories in Hong Kong since 1989 for the purpose of promoting the quality and the standards of medical laboratory technology. In the specialty of hematology and serology, one normal and one abnormal blood samples were sent to participants for measurements of Hb, RBC, MCV, Het, WBC, and Platelet every year. Fung and Tsang [5] performed tests for equality of

CV on the measurements of these six items for both the 1995 and 1996 data using the likelihood ratio test and the modified Miller asymptotic test. Both tests, which assumes normal samples, show that the CV for Hb in 1995 is significantly different from that of 1996. However, these tests do not concern us here. Using the Kolmogorov Smirnov test, the two Hb samples can well be treated as coming from lognormal populations at the 5% level as the following SPSS printouts show.

Kolmogorov-Smirnov Test

		Hb1995
N		65
Normal Parameters[a.b]	Mean	2.6836
	Std. Deviation	.01757
Most Extreme Differences	Absolute	0.88
	Positive	0.82
	Negative	– .088
Kolmogorov-Smirnov Z		.707
Asymp. Sig. (2-tailed)		.700

Kolmogorov-Smirnov Test (outlier included)

		Hb1996
N		73
Normal Parameters[a.b]	Mean	2.6924
	Std. Deviation	.02546
Most Extreme Differences	Absolute	.214
	Positive	.214
	Negative	– .150
Kolmogorov-Smirnov Z		1.828
Asymp. Sig. (2-tailed)		.002

Kolmogorov-Smirnov Test (outlier excluded)

		Hb1996
N		72
Normal Parameters[a,b]	Mean	2.6902
	Std. Deviation	.01730
Most Extreme Differences	Absolute	.155
	Positive	.109
	Negative	−.155
Kolmogorov-Smirnov Z		1.318
Asymp. Sig. (2-tailed)		.062

Hence a test for the equality of their CVs should be based on nonparametric tests instead of parametric ones. The only nonparametric test Fung and Tsang [5] used is the squared ranks test which indicates that their CVs are not significantly different. Therefore, we are interested in making inference about their common CV.

The 1995 data give the following summary measures : mean 14.64, variance 0.0665, sample size 65, and CV 0.0176. For the 1996 data with the outlier removed, the corresponding summary measures are 14.74, 0.0640, 72, and 0.0172. As Feltz and Miller [4] presented, one reasonable estimate for ψ based on all k samples is

$$\frac{\hat{\sigma}}{\mu} = \sum_{i=1}^{k} \frac{n_i}{N} \frac{s_i}{\bar{x}_i},$$

where N is the total sample size. Using this formula, ψ is estimated to be 0.0174. Based on 50,000 simulations, the proposed (approximate) 95% two-sided generalized confidence interval is found to be (0.0157 , 0.0202). The corresponding upper one-sided interval is (0, 0.0196).

6. Conclusion

Many data from clinical sciences come from lognormal instead of normal populations. In this paper, we propose an approach for making inferences about the common coefficients of variation based on several independent lognormal samples. This approach uses the concept of generalized variables. A simulation study indicates that the coverage probabilities of the proposed confidence interval and the type-I error of the proposed test are generally satisfactory. The success of the simulation approach depends largely on the accuracy of the approximations to the distribution of U, Z, and $S/\overline{X}$, the exact distributions of which are intractable when samples come from non-normal populations. The generalized variable concept in conjunction with the simulation approach might prove to be invaluable in similar situations involving other non-normal distributions such as the two-parameter exponential, gamma and uniform.

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